12 – STD – BUSINESS MATHEMATICS FORMULAE

CHAPTER 1 . APPLICATIONS OF MATRICES AND DETERMINANTS

1. Adjoint of a matrix A is $AdjA = A_c^{\,T}$.
   (where A_c is a cofactor matrix)
2. Inverse of a matrix A is $A^{-1} = \frac{1}{|A|}(AdjA)$.
3. **Results:**
   (i) $A(AdjA) = (AdjA)A = |A|I$.
   (ii) $Adj(AB) = (AdjB)(AdjA)$.
   (iii) $(AB)^{-1} = B^{-1}A^{-1}$.
   (iv) $AA^{-1} = A^{-1}A = I$.
   (v) $(A^{-1})^{-1} = A$.
4. The rank of a zero matrix (irrespective of its order) is 0.

5. Conditions for consistency of Simultaneous Linear Equations (Non – homogeneous):
   (i) If $\rho(A,B) = \rho(A) = n,$ then the equations are consistent and has unique solution.
   (ii) If $\rho(A,B) = \rho(A) < n,$ then the equations are consistent and has infinitely many solutions.
   (iii) If $\rho(A,B) \neq \rho(A),$ then the equations are inconsistent and has no solution.

6. Conditions for consistency of Simultaneous Linear Equations (Homogeneous):
   (i) If $\rho(A,B) = \rho(A) = n,$ **(OR)** If $|A| \neq 0$ then the equations have trivial solutions only.
   (ii) If $\rho(A,B) = \rho(A) < n,$ **(OR)** If $|A| = 0$ then the equations have non trivial solutions also.
7. Cramer's rule: $x = \frac{\Delta_x}{\Delta}$; $y = \frac{\Delta_y}{\Delta}$; $z = \frac{\Delta_z}{\Delta}$.
8. Technology matrix $B = \begin{pmatrix} \frac{a_{11}}{x_1} & \frac{a_{12}}{x_2} \\ \frac{a_{21}}{x_1} & \frac{a_{22}}{x_2} \end{pmatrix}$.
9. Output matrix $X = (I - B)^{-1}D$.
10. Transition Probability Matrix $T = \begin{pmatrix} P_{AA} & P_{AB} \\ P_{BA} & P_{BB} \end{pmatrix}$ **(OR)** $T = \begin{pmatrix} P_{PP} & P_{PQ} \\ P_{QP} & P_{QQ} \end{pmatrix}$
    (depends on the name of the products A, B or P, Q)

11. For finding Equilibrium share of market $A + B = 1$ **(OR)** $P + Q = 1$
    (This step carries **1 mark** and it is compulsory)

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CHAPTER 2 . ANALYTICAL GEOMETRY

1. $\frac{SP}{PM} = e.$

2. Eccentricity of parabola *e = 1.*

3. Eccentricity of ellipse *e < 1.*

4. Eccentricity of hyperbola *e > 1.*

5. Eccentricity of rectangular hyperbola $e = \sqrt{2}$..

6. Parabola:

	$y^2 = 4ax$ (opens rightward)	***$y^2 = -4ax$ (opens leftward)***	***$x^2 = 4ay$ (opens upward)***	***$x^2 = -4ay$ (opens downward)***
Vertex	(0,0)	(0,0)	(0,0)	(0,0)
Focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Latusrectum	*4a*	*4a*	*4a*	*4a*
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$

7. Ellipse:

	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad a > b$
Centre	(0,0)	(0,0)
Eccentricity	$b^2 = a^2(1-e^2)$ **(OR)** $e = \sqrt{1 - \frac{b^2}{a^2}}$	$b^2 = a^2(1-e^2)$ **(OR)** $e = \sqrt{1 - \frac{b^2}{a^2}}$
Vertices	$(a,0),(-a,0)$	$(0,a),(0,-a)$
Directrix	$x = \pm\frac{a}{e}$	$y = \pm\frac{a}{e}$
Latusrectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Foci	$(ae,0),(-ae,0)$	$(0,ae),(0,-ae)$

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8. Hyperbola:

	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Centre	(0,0)	(0,0)
Eccentricity	$b^2=a^2(e^2-1)$ **(OR)** $e=\sqrt{1+\frac{b^2}{a^2}}$	$b^2=a^2(e^2-1)$ **(OR)** $e=\sqrt{1+\frac{b^2}{a^2}}$
Vertices	$(a,0),(-a,0)$	$(0,a),(0,-a)$
Directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Latusrectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Foci	$(ae,0),(-ae,0)$	$(0,ae),(0,-ae)$

9. The general equation of Rectangular Hyperbola (R.H) is $xy = c^2$.

$\left(\text{where } c^2=\frac{a^2}{2}\right)$ (useful for objectives)

10. The eccentricity of Rectangular Hyperbola (R.H) is $e=\sqrt{2}$

CHAPTER 3 . APPLICATIONS OF DIFFERENTIATION – I

1. Average cost *(AC)* = $\frac{C}{x}(or)\frac{f(x)+k}{x}$.
2. Average variable cost *(AVC)* = $\frac{f(x)}{x}$.
3. Average fixed cost *(AFC)* = $\frac{k}{x}$.
4. Marginal cost *(MC)* = $\frac{dC}{dx}$.
5. Marginal average cost *(MAC)* = $\frac{d(AC)}{dx}$.
6. Total revenue $R = px$.
7. Average revenue *(AR)* = $\frac{R}{x}$.

   (Average revenue = Demand function i.e, $AR = p$)
8. Marginal average revenue *(MR)* = $\frac{dR}{dx}$.
9. If $x = f(p)$ is a demand function, then Elasticity of demand $\eta_d = \frac{-p}{x}.\frac{dx}{dp}$.

   (Where x – quantity demanded ; p – price)

   Note : For a demand function $q = f(p)$, $\eta_d = \frac{-p}{q}.\frac{dq}{dp}$
10. If $x = f(p)$ is a supply function, then Elasticity of supply $\eta_s = \frac{p}{x}.\frac{dx}{dp}$

    (Where x – quantity supplied ; p – price)
11. Relation between MR and Elasticity of demand is $MR = p\left(1-\frac{1}{\eta_d}\right)$.
12. At equilibrium level, $Q_d = Q_s$.
13. Equation of tangent is $(y-y_1) = m(x-x_1)$.
14. Equation of normal is $(y-y_1) = -\frac{1}{m}(x-x_1)$.

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CHAPTER 4 . APPLICATIONS OF DIFFERENTIATION – II

1. **Euler's theorem :** If u is a homogeneous function of x and y with degree n then, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu.$

   (f or z can be used in the place of u depends on the name of the function)

2. Partial Elasticities

$$\frac{Eq_1}{Ep_1}=\frac{-p_1}{q_1}\cdot\frac{\partial q_1}{\partial p_1} \quad \text{and} \quad \frac{Eq_1}{Ep_2}=\frac{-p_2}{q_1}\cdot\frac{\partial q_1}{\partial p_2}$$

3. Economic order quantity $(q_0)=\sqrt{\frac{2RC_3}{C_1}}.$

   (where R – Requirement ; C_3 – ordering cost ; C_1 – carrying cost)

4. If unit price and percentage of inventory are given then carrying cost $(C_1)=\frac{\%}{100}\times unitprice.$

5. Time between two consecutive orders $(t_0)=\frac{q_0}{R}.$

6. Number of orders = $\frac{R}{q_0}.$

7. Minimum average variable cost = $\sqrt{2RC_3C_1}.$

8. Total ordering cost = $\frac{R}{q_0}\times C_3.$

9. Total carrying cost = $\frac{q_0}{2}\times C_1.$

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CHAPTER 5 . APPLICATIONS OF INTEGRAL CALCULUS

Properties of Definite integrals:

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx.$.

2. If *f(x)* is an odd function, i.e, if *f(-x) = -f(x)* then $\int_{-a}^{a} f(x)dx = 0$..

3. If *f(x)* is an even function, i.e, if *f(-x) = f(x)* then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$..

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$..

5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx.$

6. The area under the curve $y = f(x)$, the x-axis and the ordinates at $x = a$ and $x = b$ is

   $Area = \int_a^b ydx$

7. The area under the curve $x = g(y)$, the y-axis and the lines $y = c$ and $y = d$ is

   $Area = \int_c^d xdy$.

8. If *MC* is the marginal cost function then total cost function is given by $C = \int (MC)dx + k.$

9. If *MR* is the marginal revenue function then total revenue function is given by

   $R = \int (MR)dx + k.$

10. The producers' surplus for the supply function $p = g(x)$ for the quantity x_0 and price p_0 is

    $P.S = p_0 x_0 - \int_0^{x_0} g(x)dx.$

11. The consumers' surplus for the demand function $p = f(x)$ for the quantity x_0 and price p_0 is

    $C.S = \int_0^{x_0} f(x)dx - p_0 x_0.$

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CHAPTER 6 . DIFFERENTIAL EQUATIONS

1. The General form of Homogeneous differential equations is $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$.

2. Working rule for finding the solution of linear differential equations

   (i) Extract P and Q.
   (ii) Find $\int P \; dx$.
   (iii) Find Integrating Factor (I.F) = $e^{\int P \; dx}$

3. The solution to linear differential equations of type $\frac{dy}{dx} + Py = Q$ (Where P and Q are functions of x only) is $y(I.F) = \int Q(I.F)dx + C$ **(OR)** $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C$

4. The solution to linear differential equations of type $\frac{dx}{dy} + Px = Q$ (Where P and Q are functions of y only) is $x(I.F) = \int Q(I.F)dy + C$ **(OR)** $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + C$.

5. Second order linear differential Equations

   If m_1 and m_2 are the roots of the Auxilliary equation is of the type $ax^2 + bx + c = 0$ (Quadratic equation)

   (i) If the roots m_1 and m_2 are real and distinct, C.F = $Ae^{m_1 x} + Be^{m_2 x}$.

   (ii) If the roots m_1 and m_2 are real and equal($m_1 = m_2$), C.F = $(Ax + B)e^{mx}$.

   (iii) If the roots m_1 and m_2 are unreal, i.e, if $m = \alpha \pm i\beta$, C.F = $e^{\alpha x}(A\cos\beta x + B\sin\beta x)$.
   (C.F – Complementary Function)

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CHAPTER 7 . INTERPOLATION

1. Forward operator (delta) $\Delta(y_0) = y_1 - y_0$ (or) $\Delta(f(x)) = f(x+h) - f(x)$.
2. Backward operator (nabla) $\nabla(y_1) = y_1 - y_0$ (or) $\nabla(f(x+h)) = f(x+h) - f(x)$.
3. The Shifting operator $E(y_0) = y_1,\ E^2(y_0) = y_2,\ E^3(y_0) = y_3$......... and so on.
4. The relation between forward operator (delta) and shifting operator E is

   $$\Delta = E - 1 \quad \text{(or)} \quad E = \Delta + 1.$$

5. **(For missing term problems)**
   (a) $(E-1)^3 y_0 = (E^3 - 3E^2 + 3E - 1)y_0.$

   (b) $(E-1)^4 y_0 = (E^4 - 4E^3 + 6E^2 - 4E + 1)y_0.$

   (c) $(E-1)^5 y_0 = (E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1)y_0.$

6. **Gregory – Newton's forward formula :**

   $$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 .... + \frac{u(u-1)(u-2).......(u-\overline{n-1})}{n!}\Delta^n y_0.$$

   Where $u = \frac{x - x_0}{h}$. and h – equal interval between the x - values
   (number of terms in the formula depends on the number of terms in the problem)

7. **Gregory – Newton's backward formula :**

   $$y = y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n .... + \frac{u(u+1)(u+2).......(u+\overline{n-1})}{n!}\nabla^n y_n.$$

   Where $u = \frac{x - x_n}{h}$. and h – equal interval between the x - values
   (number of terms in the formula depends on the number of terms in the problem)

8. **Lagrange's formula:**

   $$\begin{aligned} y = {} & y_0 \frac{(x-x_1)(x-x_2)..........(x-x_n)}{(x_0-x_1)(x_0-x_2)..........(x_0-x_n)} \\ & + y_1 \frac{(x-x_0)(x-x_2)..........(x-x_n)}{(x_1-x_0)(x_1-x_2)..........(x_1-x_n)} \\ & + ... \\ & + y_n \frac{(x-x_0)(x-x_1)..........(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)..........(x_n-x_{n-1})} \end{aligned}$$

   (depends on the number of terms given in the problem)

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9. **Line Of Best Fit:**

Normal equations are

$$a\sum x + nb = \sum y$$
$$a\sum x^2 + b\sum x = \sum xy$$

The line of best fit is

$$y = ax + b$$

CHAPTER 8 . PROBABILITY DISTRIBUTION

1. If X is a continuous random variable, then $P(a < X < b) = \int_a^b f(x)dx.$
2. For a discrete random variable X,

   Mean $E(X) = \sum x_i p_i.$

   $$E(X^2) = \sum x_i^2 p_i.$$
   $$Var(X) = E(X^2) - [E(X)]^2.$$
3. For a continuous random variable X,

   Mean $E(X) = \int_{-\infty}^{\infty} xf(x)dx.$

   $$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx.$$
   $$Var(X) = E(X^2) - [E(X)]^2.$$
4. If the discrete random variable X follows Binomial distribution then

   $$P(X = x) = nC_x p^x q^{n-x}, \quad x = 0,1,2,.........\, .n.$$
5. Results related to Binomial distribution:

   Mean = np ; Variance = npq ; *and* $p + q = 1$
6. If the discrete random variable X follows Poisson distribution then

   $$P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0,1,2,.........\, ..$$
7. Results related to Poisson distribution:

   Mean $(\lambda) = np$; Variance = λ.

   In Poisson distribution **Mean = Variance**

8. If the continuous random X follows Normal distribution, then its p.d.f is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty < x < \infty.$$

9. To convert Normal variate X to standard Normal variate *z* we use, $z = \frac{X-\mu}{\sigma}$.

CHAPTER 9 . SAMPLING DISTRIBUTION

1. **Notations:**
   - (a) N – Population size
   - (b) n – Sample size
   - (c) $\overline{X}$ – Mean of the sample
   - (d) μ – Mean of the population
   - (e) s - Standard deviation (S.D) of sample
   - (f) σ - Standard deviation (S.D) of population

2. Confidence limits for $\mu = \overline{X} \pm (Z_c)\frac{s}{\sqrt{n}}$. (If N is not given)

$$= \overline{X} \pm (Z_c)\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}. \text{ (If } N \text{ is given)}$$

3. Confidence intervals for proportion $= p \pm (Z_c)\sqrt{\frac{pq}{n}}$. (If N is not given)

$$= p \pm (Z_c)\sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}}. \text{ (If } N \text{ is given)}$$

Note : For 95% confidence interval $Z_c = 1.96$

For 99% confidence interval $Z_c = 2.58$

4. **Testing of Hypothesis Formulae:**

Test statistic $Z = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$.

Test statistic $Z = \frac{p-P}{\sqrt{\frac{pq}{n}}}$.

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5. For 5% level of significance : Acceptance region $|Z| < 1.96.$

Critical region $|Z| \geq 1.96.$

6. For 1% level of significance : Acceptance region $|Z| < 2.58.$

Critical region $|Z| \geq 2.58.$

CHAPTER 10 . APPLIED STATISTICS

1. **Correlation coefficient formulae:**

(a) $r(X,Y) = \dfrac{N\sum XY - \sum X \sum Y}{\sqrt{N\sum X^2 - \left(\sum X\right)^2}\sqrt{N\sum Y^2 - \left(\sum Y\right)^2}}$

(If $\overline{X}, \overline{Y}$ are integers or non-integers)

(b) $r(x,y) = \dfrac{\sum xy}{\sqrt{\sum x^2}\sqrt{\sum y^2}}$ Where $x = X - \overline{X}$ and $y = Y - \overline{Y}$.

(If $\overline{X}, \overline{Y}$ are integers) and $\overline{X} = \dfrac{\sum X}{n}$ and $\overline{Y} = \dfrac{\sum Y}{n}$

(c) $r(X,Y) = \dfrac{N\sum dxdy - \sum dx \sum dy}{\sqrt{N\sum dx^2 - \left(\sum dx\right)^2}\sqrt{N\sum dy^2 - \left(\sum dy\right)^2}}$

(If $\overline{X}, \overline{Y}$ are integers or non - integers)

Where $dx = X - A$ and $dy = Y - B$. (A, B are arbitrary values of X and Y)

(**Note:** Correlation coefficient should lie between -1 and 1)

2. **Regression Formulae:**

(a) Regression line of X on Y is

$$(X - \overline{X}) = b_{xy}(Y - \overline{Y}).$$

(b) Regression line of Y on X is

$(Y - \overline{Y}) = b_{yx}(X - \overline{X})$. Where $\overline{X} = \dfrac{\sum X}{n}$ and $\overline{Y} = \dfrac{\sum Y}{n}$

Where $b_{xy} = \dfrac{N\sum XY - \sum X \sum Y}{\sum Y^2 - \left(\sum Y\right)^2}$ and $b_{yx} = \dfrac{N\sum XY - \sum X \sum Y}{\sum X^2 - \left(\sum X\right)^2}$

(If $\overline{X}, \overline{Y}$ are integers or non - integers)

Where $b_{xy} = \dfrac{\sum xy}{\sum y^2}$ and $b_{yx} = \dfrac{\sum xy}{\sum x^2}$ (If $\overline{X}, \overline{Y}$ are integers)

(**Note:** Regression lines will intersect at $(\overline{X}, \overline{Y})$)

3. Seasonal Index = $\dfrac{Quaterly \quad average}{Grand \quad average} \times 100$.

4. Index Numbers:

(a) Laspeyre's price Index number $\left(P_{01}{}^{L}\right) = \dfrac{\sum p_1 q_0}{\sum p_0 q_0} \times 100.$

(b) Paasche's price index number $\left(P_{01}{}^{P}\right) = \dfrac{\sum p_1 q_1}{\sum p_0 q_1} \times 100.$

(c) Fisher's price index number $\left(P_{01}{}^{F}\right) = \sqrt{\dfrac{\sum p_1 q_0}{\sum p_0 q_0} \times \dfrac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100.$

(OR) $\left(P_{01}{}^{F}\right) = \sqrt{P_{01}{}^{L} \times P_{01}{}^{P}}.$

(d) **Cost of Living Index numbers:**

(i) Aggregate Expenditure method *(C.L.I)* $= \dfrac{\sum p_1 q_0}{\sum p_0 q_0} \times 100.$

(ii) Family Budget method *(C.L.I)* $= \dfrac{\sum PV}{\sum V}.$

Where $P = \dfrac{p_1}{p_0} \times 100$ and $V = p_0 q_0$.

5. Statistical Quality Control *(SQC)* Formulae:

Range chart (*R* Chart): $C.L = \overline{R} = \dfrac{\sum R}{n}.$

$U.C.L = D_4 \overline{R}$

$L.C.L = D_3 \overline{R}$

$\overline{X}$ Chart : $C.L = \overline{\overline{X}} = \dfrac{\sum \overline{X}}{n}.$

$U.C.L = \overline{\overline{X}} + A_2 \overline{R}.$

$L.C.L = \overline{\overline{X}} - A_2 \overline{R}.$

(Where *C.L* – Central Line ; *U.C.L* - Upper Control Line ; *L.C.L* - Lower Control Line)

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