CHAPTER 1 . APPLICATIONS OF MATRICES AND DETERMINANTS

1. Adjoint of a matrix $A$ is $\text{Adj}A = A^T$.
   (where $A_c$ is a cofactor matrix)

2. Inverse of a matrix $A$ is $A^{-1} = \frac{1}{|A|}(\text{Adj}A)$.

3. Results:
   (i) $A(\text{Adj}A) = (\text{Adj}A)A = |A|I$.
   (ii) $\text{Adj}(AB) = (\text{Adj}B)(\text{Adj}A)$.
   (iii) $(AB)^{-1} = B^{-1}A^{-1}$.
   (iv) $AA^{-1} = A^{-1}A = I$.
   (v) $(A^{-1})^{-1} = A$.

4. The rank of a zero matrix (irrespective of its order) is 0.

5. Conditions for consistency of Simultaneous Linear Equations (Non – homogeneous):
   (i) If $\rho(A, B) = \rho(A) = n$, then the equations are consistent and has unique solution.
   (ii) If $\rho(A, B) = \rho(A) < n$, then the equations are consistent and has infinitely many solutions.
   (iii) If $\rho(A, B) \neq \rho(A)$, then the equations are inconsistent and has no solution.

6. Conditions for consistency of Simultaneous Linear Equations (Homogeneous):
   (i) If $\rho(A, B) = \rho(A) = n$, (OR) If $|A| \neq 0$ then the equations have trivial solutions only.
   (ii) If $\rho(A, B) = \rho(A) < n$, (OR) If $|A| = 0$ then the equations have non trivial solutions also.

7. Cramer’s rule: $x = \frac{\Delta_x}{\Delta}; \quad y = \frac{\Delta_y}{\Delta}; \quad z = \frac{\Delta_z}{\Delta}$.

8. Technology matrix $B = \begin{bmatrix} a_{11} & a_{12} \\ x_1 & x_2 \\ a_{21} & a_{22} \\ x_1 & x_2 \end{bmatrix}$.

9. Output matrix $X = (I - B)^{-1}D$.

10. Transition Probability Matrix $T = \begin{bmatrix} P_{AA} & P_{AB} \\ P_{RA} & P_{BB} \end{bmatrix}$ (OR) $T = \begin{bmatrix} P_{PP} & P_{PQ} \\ P_{QP} & P_{QQ} \end{bmatrix}$
    (depends on the name of the products $A$, $B$ or $P$, $Q$)

11. For finding Equilibrium share of market $A + B = 1$ (OR) $P + Q = 1$
    (This step carries 1 mark and it is compulsory)
CHAPTER 2 . ANALYTICAL GEOMETRY

1. \( \frac{SP}{PM} = e \).

2. Eccentricity of parabola \( e = 1 \).

3. Eccentricity of ellipse \( e < 1 \).

4. Eccentricity of hyperbola \( e > 1 \).

5. Eccentricity of rectangular hyperbola \( e = \sqrt{2} .. \)

6. Parabola:

<table>
<thead>
<tr>
<th></th>
<th>( y^2 = 4ax ) (opens rightward)</th>
<th>( y^2 = -4ax ) (opens leftward)</th>
<th>( x^2 = 4ay ) (opens upward)</th>
<th>( x^2 = -4ay ) (opens downward)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Focus</td>
<td>((a,0))</td>
<td>((-a,0))</td>
<td>((0,a))</td>
<td>((0,-a))</td>
</tr>
<tr>
<td>Directrix</td>
<td>( x = -a )</td>
<td>( x = a )</td>
<td>( y = -a )</td>
<td>( y = a )</td>
</tr>
<tr>
<td>Latusrectum</td>
<td>( 4a )</td>
<td>( 4a )</td>
<td>( 4a )</td>
<td>( 4a )</td>
</tr>
<tr>
<td>Axis</td>
<td>( y = 0 )</td>
<td>( y = 0 )</td>
<td>( x = 0 )</td>
<td>( x = 0 )</td>
</tr>
</tbody>
</table>

7. Ellipse:

<table>
<thead>
<tr>
<th>( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 ), ( a &gt; b )</th>
<th>( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 ), ( a &gt; b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>( b^2 = a^2(1-e^2) ) (OR)</td>
</tr>
<tr>
<td>( e = \sqrt{1 - \frac{b^2}{a^2}} )</td>
<td>( e = \sqrt{1 - \frac{b^2}{a^2}} )</td>
</tr>
<tr>
<td>Vertices</td>
<td>((a,0),(-a,0))</td>
</tr>
<tr>
<td>Directrix</td>
<td>( x = \pm \frac{a}{e} )</td>
</tr>
<tr>
<td>Latusrectum</td>
<td>( \frac{2b^2}{a} )</td>
</tr>
<tr>
<td>Foci</td>
<td>((ae,0),(-ae,0))</td>
</tr>
</tbody>
</table>

8. **Hyperbola:**

<table>
<thead>
<tr>
<th></th>
<th>( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 )</th>
<th>( \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Centre</strong></td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td><strong>Eccentricity</strong></td>
<td>( b^2 = a^2(e^2 - 1) ) (OR) ( b^2 = a^2(e^2 - 1) )</td>
<td>( e = \sqrt{1 + \frac{b^2}{a^2}} ) (OR) ( e = \sqrt{1 + \frac{b^2}{a^2}} )</td>
</tr>
<tr>
<td><strong>Vertices</strong></td>
<td>((a,0),(-a,0))</td>
<td>((0,a),(0,-a))</td>
</tr>
<tr>
<td><strong>Directrix</strong></td>
<td>( x = \pm \frac{a}{e} )</td>
<td>( y = \pm \frac{a}{e} )</td>
</tr>
<tr>
<td><strong>Latusrectum</strong></td>
<td>( \frac{2b^2}{a} )</td>
<td>( \frac{2b^2}{a} )</td>
</tr>
<tr>
<td><strong>Foci</strong></td>
<td>((ae,0),(-ae,0))</td>
<td>((0,ae),(0,-ae))</td>
</tr>
</tbody>
</table>

9. The general equation of Rectangular Hyperbola (R.H) is \( xy = c^2 \).

\[
\left( \text{where } c^2 = \frac{a^2}{2} \right)
\]

(useful for objectives)

10. The eccentricity of Rectangular Hyperbola (R.H) is \( e = \sqrt{2} \)
CHAPTER 3. APPLICATIONS OF DIFFERENTIATION – I

1. Average cost \( (AC) = \frac{C}{x} \left( or \frac{f(x) + k}{x} \right) \).

2. Average variable cost \( (AVC) = \frac{f(x)}{x} \).

3. Average fixed cost \( (AFC) = \frac{k}{x} \).

4. Marginal cost \( (MC) = \frac{dC}{dx} \).

5. Marginal average cost \( (MAC) = \frac{d(AC)}{dx} \).

6. Total revenue \( R = px \).

7. Average revenue \( (AR) = \frac{R}{x} \).

   (Average revenue = Demand function i.e, \( AR = p \))

8. Marginal average revenue \( (MR) = \frac{dR}{dx} \).

9. If \( x = f(p) \) is a demand function, then Elasticity of demand \( \eta_d = -\frac{p}{x} \cdot \frac{dx}{dp} \).

   (Where \( x \) – quantity demanded ; \( p \) – price)

   Note: For a demand function \( q = f(p) \), \( \eta_d = \frac{p}{q} \cdot \frac{dq}{dp} \)

10. If \( x = f(p) \) is a supply function, then Elasticity of supply \( \eta_s = \frac{p}{x} \cdot \frac{dx}{dp} \).

    (Where \( x \) – quantity supplied ; \( p \) – price)

11. Relation between \( MR \) and Elasticity of demand is \( MR = p \left( 1 - \frac{1}{\eta_d} \right) \).

12. At equilibrium level, \( Q_d = Q_s \).

13. Equation of tangent is \( (y - y_1) = m(x - x_1) \)

14. Equation of normal is \( (y - y_1) = -\frac{1}{m}(x - x_1) \)

CHAPTER 4 . APPLICATIONS OF DIFFERENTIATION – II

1. **Euler’s theorem**: If \( u \) is a homogeneous function of \( x \) and \( y \) with degree \( n \) then, \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu. \)  
   
   (\( f \) or \( z \) can be used in the place of \( u \) depends on the name of the function)

2. Partial Elasticities
   \[
   \frac{Ep_1}{Ep} = -\frac{p_1}{q_1} \frac{\partial q_1}{\partial p_1} \quad \text{and} \quad \frac{Ep_2}{Ep} = -\frac{p_2}{q_1} \frac{\partial q_1}{\partial p_2}
   \]

3. Economic order quantity \((q_0) = \sqrt{\frac{2RC_3}{C_1}}.\)
   (where \( R \) – Requirement ; \( C_3 \) – ordering cost ; \( C_1 \) – carrying cost)

4. If unit price and percentage of inventory are given then carrying cost \( (C_1) = \frac{\%}{100} \times \text{unitprice}.\)

5. Time between two consecutive orders \((t_0) = \frac{q_0}{R}.\)

6. Number of orders = \( \frac{R}{q_0}.\)

7. Minimum average variable cost = \( \sqrt{2RC_3C_1}.\)

8. Total ordering cost = \( \frac{R}{q_0} \times C_3.\)

9. Total carrying cost = \( \frac{q_0}{2} \times C_1.\)

CHAPTER 5. APPLICATIONS OF INTEGRAL CALCULUS

Properties of Definite integrals:

1. \[ \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx. \]

2. If \( f(x) \) is an odd function, i.e, if \( f(-x) = -f(x) \) then \[ \int_{-a}^{a} f(x)dx = 0. \]

3. If \( f(x) \) is an even function, i.e, if \( f(-x) = f(x) \) then \[ \int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx. \]

4. \[ \int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx. \]

5. \[ \int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx. \]

6. The area under the curve \( y = f(x) \), the x-axis and the ordinates at \( x = a \) and \( x = b \) is \[ Area = \int_{a}^{b} ydx. \]

7. The area under the curve \( x = g(y) \), the y-axis and the lines \( y = c \) and \( y = d \) is \[ Area = \int_{c}^{d} xdy. \]

8. If \( MC \) is the marginal cost function then total cost function is given by \[ C = \int (MC)dx + k. \]

9. If \( MR \) is the marginal revenue function then total revenue function is given by \[ R = \int (MR)dx + k. \]

10. The producers’ surplus for the supply function \( p = g(x) \) for the quantity \( x_0 \) and price \( p_0 \) is \[ P.S = p_0x_0 - \int_{0}^{x_0} g(x)dx. \]

11. The consumers’ surplus for the demand function \( p = f(x) \) for the quantity \( x_0 \) and price \( p_0 \) is \[ C.S = \int_{0}^{x_0} f(x)dx - p_0x_0. \]
CHAPTER 6. DIFFERENTIAL EQUATIONS

1. The General form of Homogeneous differential equations is \( \frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}. \)

2. Working rule for finding the solution of linear differential equations
   
   (i) Extract P and Q.
   
   (ii) Find \( \int P \, dx. \)
   
   (iii) Find Integrating Factor (I.F) = \( e^{\int P \, dx}. \)

3. The solution to linear differential equations of type \( \frac{dy}{dx} + P(x) = Q(x) \) (Where \( P \) and \( Q \) are functions of \( x \) only) is \( y(I.F) = \int Q(I.F) \, dx + C \) (OR) \( ye^{\int P \, dx} = \int Qe^{\int P \, dx} \, dx + C \)

4. The solution to linear differential equations of type \( \frac{dx}{dy} + P(y) = Q(y) \) (Where \( P \) and \( Q \) are functions of \( y \) only) is \( x(I.F) = \int Q(I.F) \, dy + C \) (OR) \( xe^{\int P \, dy} = \int Qe^{\int P \, dy} \, dy + C. \)

5. Second order linear differential Equations
   
   If \( m_1 \) and \( m_2 \) are the roots of the Auxilliary equation is of the type \( ax^2 + bx + c = 0 \) (Quadratic equation)
   
   (i) If the roots \( m_1 \) and \( m_2 \) are real and distinct, C.F = \( Ae^{m_1x} + Be^{m_2x}. \)
   
   (ii) If the roots \( m_1 \) and \( m_2 \) are real and equal\((m_1 = m_2)\), C.F = \( (Ax + B)e^{mx}. \)
   
   (iii) If the roots \( m_1 \) and \( m_2 \) are unreal, i.e, if \( m = \alpha \pm i\beta \), C.F = \( e^{\alpha x}(A\cos \beta x + B\sin \beta x). \)
   
   (C.F – Complementary Function)
CHAPTER 7. INTERPOLATION

1. Forward operator (delta) \( \Delta(y_0) = y_1 - y_0 \) (or) \( \Delta(f(x)) = f(x + h) - f(x) \).

2. Backward operator (nabla) \( \nabla(y_1) = y_1 - y_0 \) (or) \( \nabla(f(x + h)) = f(x + h) - f(x) \).

3. The Shifting operator \( E(y_0) = y_1, \ E^2(y_0) = y_2, \ E^3(y_0) = y_3 \ldots \) and so on.

4. The relation between forward operator (delta) and shifting operator \( E \) is

\[
\Delta = E - 1 \quad \text{(or)} \quad E = \Delta + 1.
\]

5. (For missing term problems)
   (a) \( (E - 1)^3 y_0 = (E^3 - 3E^2 + 3E - 1)y_0 \).
   (b) \( (E - 1)^4 y_0 = (E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 \).
   (c) \( (E - 1)^5 y_0 = (E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1)y_0 \).

6. Gregory – Newton’s forward formula:

\[
y = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u - 1)}{2!} \Delta^2 y_0 + \frac{u(u - 1)(u - 2)}{3!} \Delta^3 y_0 + \ldots + \frac{u(u - 1)(u - 2)\ldots(u - n + 1)}{n!} \Delta^n y_0.
\]

Where \( u = \frac{x - x_0}{h} \). and \( h \) – equal interval between the \( x \)-values

(number of terms in the formula depends on the number of terms in the problem)

7. Gregory – Newton’s backward formula:

\[
y = y_n + \frac{u}{1!} \nabla y_n + \frac{u(u + 1)}{2!} \nabla^2 y_n + \frac{u(u + 1)(u + 2)}{3!} \nabla^3 y_n + \ldots + \frac{u(u + 1)(u + 2)\ldots(u + n - 1)}{n!} \nabla^n y_n.
\]

Where \( u = \frac{x - x_n}{h} \). and \( h \) – equal interval between the \( x \)-values

(number of terms in the formula depends on the number of terms in the problem)

8. Lagrange’s formula:

\[
y = y_0 \frac{(x - x_1)(x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)} + y_1 \frac{(x - x_0)(x - x_2)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots(x_1 - x_n)} + \ldots + y_n \frac{(x - x_0)(x - x_1)\ldots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)\ldots(x_n - x_{n-1})}
\]

(depending on the number of terms given in the problem)

9. **Line Of Best Fit:**

Normal equations are

\[ a \sum x + nb = \sum y \]
\[ a \sum x^2 + b \sum x = \sum xy \]

The line of best fit is

\[ y = ax + b \]

**CHAPTER 8 . PROBABILITY DISTRIBUTION**

1. If \( X \) is a continuous random variable, then \( P(a < X < b) = \int_a^b f(x)dx \).

2. For a discrete random variable \( X \),

   Mean \( E(X) = \sum x_i p_i \).

   \( E(X^2) = \sum x_i^2 p_i \).

   \( Var(X) = E(X^2) - [E(X)]^2 \).

3. For a continuous random variable \( X \),

   Mean \( E(X) = \int_{-\infty}^\infty x f(x)dx \).

   \( E(X^2) = \int_{-\infty}^\infty x^2 f(x)dx \).

   \( Var(X) = E(X^2) - [E(X)]^2 \).

4. If the discrete random variable \( X \) follows Binomial distribution then

   \( P(X = x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0,1,2, \ldots, n \).

5. Results related to Binomial distribution:

   Mean = \( np \);  Variance = \( npq \);  and  \( p + q = 1 \).

6. If the discrete random variable \( X \) follows Poisson distribution then

   \( P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0,1,2, \ldots \).

7. Results related to Poisson distribution:

   Mean \( \lambda = np \);  Variance = \( \lambda \).

In Poisson distribution \textbf{Mean = Variance}
8. If the continuous random $X$ follows Normal distribution, then its p.d.f is given by
\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty. \]

9. To convert Normal variate $X$ to standard Normal variate $z$ we use, $z = \frac{X - \mu}{\sigma}$.

**CHAPTER 9 . SAMPLING DISTRIBUTION**

1. **Notations:**
   (a) $N$ – Population size
   (b) $n$ – Sample size
   (c) $\overline{X}$ – Mean of the sample
   (d) $\mu$ – Mean of the population
   (e) $s$ - Standard deviation (S.D) of sample
   (f) $\sigma$ - Standard deviation (S.D) of population

2. Confidence limits for $\mu = \overline{X} \pm (Z_c)\frac{s}{\sqrt{n}}$. (If $N$ is not given)
   \[ = \overline{X} \pm (Z_c)\frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}. \text{ (If } N \text{ is given)} \]

3. Confidence intervals for proportion $p = \overline{X} \pm (Z_c)\sqrt{\frac{pq}{n}}$. (If $N$ is not given)
   \[ = \overline{X} \pm (Z_c)\sqrt{\frac{pq}{n}} \sqrt{\frac{N-n}{N-1}}. \text{ (If } N \text{ is given)} \]
   **Note**: For 95% confidence interval $Z_c = 1.96$

   For 99% confidence interval $Z_c = 2.58$

4. **Testing of Hypothesis Formulae:**
   Test statistic $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$.
   Test statistic $Z = \frac{p - P}{\sqrt{\frac{pq}{n}}}$.

**K. MANIMARAN. M.Sc.,B.Ed ; P.G. Asst - GOLDEN GATES MAT. HR. SEC. SCHOOL, SALEM – 8. PH : 94899 69230.**
5. For 5% level of significance: Acceptance region $|Z| < 1.96$.

Critical region $|Z| \geq 1.96$.

6. For 1% level of significance: Acceptance region $|Z| < 2.58$.

Critical region $|Z| \geq 2.58$.

CHAPTER 10. APPLIED STATISTICS

1. Correlation coefficient formulae:

(a) $r(X, Y) = \frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}}$

(If $\bar{X}, \bar{Y}$ are integers or non-integers)

(b) $r(x, y) = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$ Where $x = X - \bar{X}$ and $y = Y - \bar{Y}$.

(If $\bar{X}, \bar{Y}$ are integers) and $\bar{X} = \frac{\sum X}{n}$ and $\bar{Y} = \frac{\sum Y}{n}$

(c) $r(X, Y) = \frac{N \sum dx dy - \sum dx \sum dy}{\sqrt{N \sum dx^2 - (\sum dx)^2} \sqrt{N \sum dy^2 - (\sum dy)^2}}$

(If $\bar{X}, \bar{Y}$ are integers or non-integer)

Where $dx = X - A$ and $dy = Y - B$. ($A, B$ are arbitrary values of $X$ and $Y$)

(Note: Correlation coefficient should lie between -1 and 1)

2. Regression Formulae:

(a) Regression line of $X$ on $Y$ is

$(X - \bar{X}) = b_{xy} (Y - \bar{Y})$.

(b) Regression line of $Y$ on $X$ is

$(Y - \bar{Y}) = b_{yx} (X - \bar{X})$. Where $\bar{X} = \frac{\sum X}{n}$ and $\bar{Y} = \frac{\sum Y}{n}$

Where $b_{xy} = \frac{N \sum XY - \sum X \sum Y}{\sum Y^2 - (\sum Y)^2}$ and $b_{yx} = \frac{N \sum XY - \sum X \sum Y}{\sum X^2 - (\sum X)^2}$

(If $\bar{X}, \bar{Y}$ are integers or non-integers)

Where $b_{xy} = \frac{\sum xy}{\sum y^2}$ and $b_{yx} = \frac{\sum xy}{\sum x^2}$ (If $\bar{X}, \bar{Y}$ are integers)

(Note: Regression lines will intersect at \((X, Y)\)).

3. Seasonal Index = \(\frac{\text{Quarterly average}}{\text{Grand average}} \times 100\).

4. Index Numbers:
   
   (a) Laspeyre’s price Index number \(\left( P_{01}^L \right) = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100\).

   (b) Paasche’s price index number \(\left( P_{01}^P \right) = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100\).

   (c) Fisher’s price index number \(\left( P_{01}^F \right) = \sqrt{\frac{\sum p_1 q_0 \sum p_1 q_1}{\sum p_0 q_0 \sum p_0 q_1}} \times 100\).

   (OR) \(\left( P_{01}^F \right) = \sqrt{P_{01}^L \times P_{01}^P}\).

   (d) Cost of Living Index numbers:

   (i) Aggregate Expenditure method \((C.L.I) = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100\).

   (ii) Family Budget method \((C.L.I) = \frac{\sum PV}{\sum V}\).

   Where \(P = \frac{p_1}{p_0} \times 100\) and \(V = p_0 q_0\).

5. Statistical Quality Control (SQC) Formulae:

   Range chart \((R\text{ Chart})\):

   \[ C.L. = \bar{R} = \frac{\sum R}{n} \]

   \[ U.C.L. = D_4 \bar{R} \]

   \[ L.C.L. = D_3 \bar{R} \]

   \(\bar{X}\text{ Chart}:

   \[ C.L. = \bar{X} = \frac{\sum X}{n} \]

   \[ U.C.L. = \bar{X} + A_2 \bar{R} \]

   \[ L.C.L. = \bar{X} - A_2 \bar{R} \]

   (Where \(C.L.\) – Central Line ; \(U.C.L\) - Upper Control Line ; \(L.C.L\) - Lower Control Line)