

**THIRU TUITION CENTRE**

KUNICHI, TIRUPATTUR, VELLORE DISTRICT.

XII STD Chapters 1-10 important questions March 26, 2012

**1. Applications of Matrices and Determinants**

1. Find the adjoint matrix  $\begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

2. Find the adjoint matrix  $\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$  verify the result

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$

3. Find the rank matrix  $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

4. Find the rank matrix  $\begin{pmatrix} 3 & 1 & 2 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 1 & 3 & 0 \end{pmatrix}$

5. Show that the adjoint of  $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$  is  $3A^T$ .

6. Solve by inversion matrix method

$$7x + 3y = -1, 2x + y = 0. \quad (\text{ii}) \quad x + y = 3, 2x + 3y = 8$$

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7. Solve the following non-homogeneous equations of three unknowns by using determinants

$$2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4.$$

8. Solve the following non-homogeneous equations of three unknowns by using determinants

$$3x + y - z = 2, 2x - y + 2z = 6, 2x + y - 2z = -2.$$

9. Examine the consistency of the following system of equations. If it is consistent, then solve the same

$$x + y + z = 7, x + 2y + 3z = 18, y + 2z = 6$$

10. Solve the following non-homogeneous equations of three unknowns by using determinants

$$4x + 5y = 9, 8x + 10y = 18$$

## 2. Vector Algebra

11. If the sum of two unit vectors is a unit vector prove that the magnitude of their difference is  $\sqrt{3}$

12. Show that the points whose position vectors

$$4\vec{i} - 3\vec{j} + \vec{k}, 2\vec{i} - 4\vec{j} + 5\vec{k}, \vec{i} - \vec{j}$$

13. Find the area of the triangle whose vertices are  $(3, -1, 2)$ ,  $(1, -1, -3)$ , and  $(4, -3, 1)$

14. Find the vector and cartesian equation of the line joining the points  $(1, -2, 1)$  and  $(0, -2, 3)$

15. (i) Find the angle between the lines

$$\vec{r} = 5\vec{i} - 7\vec{j} + \mu \left( -\vec{i} + 4\vec{j} + 2\vec{k} \right)$$

- (ii) Find the angle between the line  $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$  and the plane  $3x + 4y + z + 5 = 0$

16. If  $A(-1, 4, -3)$  is one end of a diameter AB of the sphere  $x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0$ , then find the coordinates of B.
17. Show that the lines  $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$  and  $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$  intersect find their point of intersection.
18. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares sides.
19. Forces of magnitudes 3 and 4 units acting in the directions  $6\vec{i} + 2\vec{j} + 3\vec{k}$  and  $3\vec{i} - 2\vec{j} + 6\vec{k}$  respectively act on a particle which is displaced from the point  $(2, 2, -1)$  to  $4, 3, 1$ . Find the work done by the forces.
20. P.T.  $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

### 3. Complex Numbers

21. For what values of  $x$  and  $y$ , the numbers  $-3 + i3x^2y$  and  $x^2 + y + 4i$  complex conjugate of each other?
22. For any two complex numbers  $z_1$  and  $z_2$
- (i)  $|z_1 z_2| = |z_1| \cdot |z_2|$  (ii)  $\arg(z_1/z_2) = \arg(z_1) + \arg(z_2)$
23. Find the modulus and argument of the following complex numbers : (i)  $-1 - i\sqrt{3}$  (ii)  $2 + i2\sqrt{3}$
24.  $P$  represents the variable complex number  $z$ . Find the locus of  $P$ , if (i)  $\operatorname{Re}\left(\frac{z-1}{z+i}\right) = 1$  (ii)  $\operatorname{Im}\left[\frac{2z+1}{iz+1}\right] = -2$
25. If  $\arg(z-1) = \frac{\pi}{2}$  and  $\arg(z+1) = \frac{2\pi}{3}$  then P.T  $|z| = 1$
26. Solve the equation  $6x^4 - 25x^3 + 32x^2 + 3x - 10 = 0$  given that one of the roots is  $2 - i$

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27. If  $n$  is a positive integer

$$\text{P.T } (\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos\left(\frac{n\pi}{6}\right)$$

28. If  $x + \frac{1}{x} = 2\cos\theta$  then P.T

$$\text{(i) } x^n + \frac{1}{x^n} = 2\cos n\theta \quad \text{(ii) } x^n - \frac{1}{x^n} = 2i\sin n\theta$$

29. (i) Find the  $n$ th roots of unity. verify that G.P

$$\text{(ii) Solve the equation } x^9 + x^5 - x^4 - 1 = 0$$

30. Find all the values of  $(\sqrt{3} + i)^{\frac{2}{3}}$

#### 4. Analytical Geometry

31. The headlight of a motor vehicle is a parabolic reflector of diameter 12cm and depth 4cm. Find the position of bulb on the axis of the reflector for effective functioning of the headlight.

32. The focus of a parabolic mirror is at a distance of 8cm from its centre (vertex). If the mirror is 25 cm deep, find the diameter of the mirror.

33. Find the equation of the ellipse given that the centre is  $(4, -1)$ , focus is  $(1, -1)$  and passing through  $(8, 0)$ .

34. A kho-kho player in a practice session while running realises that the sum of the distances from the two kho-kho poles from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

35. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun? (ii) the greatest possible distance between mercury and sun.
36. Find the equation of the locus of all points such that the differences of their distances from  $(4, 0)$  and  $(-4, 0)$  is always equal to 2.
37. Show that the locus of a point which moves so that the difference of its distances from the points  $(5, 0)$  and  $(-5, 0)$  is 8 is  $9x^2 - 16y^2 = 144$ .
38. Find the equations of the two tangents that can be drawn from the point  $(5, 2)$  to the ellipse  $2x^2 + 7y^2 = 14$ .
39. Show that the line  $x - y + 4 = 0$  is a tangent to the ellipse  $x^2 + 3y^2 = 12$ . Find the co-ordinates of the point of contact.
40. Find the equation of the hyperbola which passes through the point  $(2, 3)$  and has the asymptotes  $4x + 3y - 7 = 0$  and  $x - 2y = 1$ .
41. Show that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact.
42. Find the equation of the rectangular hyperbola which has its centre at  $(2, 1)$ , one of its asymptotes  $3x - y - 5 = 0$  and which passes through the point  $(1, -1)$

### 5. Differential Calculus Applications - I

43. A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/sec how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 m from the wall?
44. The altitude of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of 2 cm<sup>2</sup>/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100cm<sup>2</sup>.
45. Find the equations of those tangents to the circle  $x^2 + y^2 = 52$ , which are parallel to the straight line  $2x + 3y = 6$ .
46. Apply Rolles theorem to find points on curve  $y = 1 + \cos x$ , where the tangent is parallel to x-axis in  $[0, 2\pi]$
47. A cylindrical hole 4 mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12 mm. Estimate the amount of metal removed.
48. Obtain the Taylors series expansion of  $f(x) = \sin x$  about  $x = \frac{\pi}{2}$
49. Obtain the Maclaurins Series expansion for  $\tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$
50. Evaluate:  $\lim_{n \rightarrow \infty} x^{\sin x}$
51. Determine for which values of  $x$ , the function  $f(x) = 2x^3 - 15x^2 + 36x + 1$  is increasing and for which it is decreasing. Also determine the points where the tangents to the graph of the function are parallel to the x axis.

52. Find the absolute maximum and minimum values of the function  $y = x^3 - 3x^2 + 1$ ,  $-\frac{1}{2} \leq x \leq 4$
53. Find a point on the parabola  $y^2 = 2x$  that is closest to the point  $(1, 4)$
54. Find two positive numbers whose product is 100 and whose sum is minimum.
55. Find the points of inflection and determine the intervals of convexity and concavity of the Gaussian curve  $y = e^{-x^2}$
56. Find the intervals of concavity and the points of inflection of the following function
- $$f(x) = 2x^3 + 5x^2 - 4x$$

## 6. Differential Calculus Applications - II

57. Use differentials to find an approximate value for  
(i)  $\sqrt[3]{65}$  (ii)  $y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$
58. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error in computing (i) the volume of the cube and (ii) the surface area of cube.
59. The radius of a circular disc is given as 24 cm with a maximum error in measurement of 0.02 cm. (i) Use differentials to estimate the maximum error in the calculated area of the disc. (ii) Compute the relative error ?
60. The time of swing  $T$  of a pendulum is given by  $T = k\sqrt{l}$  where  $k$  is a constant. Determine the percentage error in the time of swing if the length of the pendulum  $l$  changes from 32.1 cm to 32.0 cm.

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61. Discuss the following curves for (i) existence (ii) symmetry  
(iii) asymptotes (iv) loops

$$y^2(2+x) = x^2(6-x)$$

62. If  $u = \log(\tan x + \tan y + \tan z)$  then P.T  $\sum \sin 2x \frac{\partial u}{\partial x} = 2$

63. Using Eulers theorem, prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$  if

$$u = \sin^{-1} \left( \frac{x-y}{\sqrt{x}+\sqrt{y}} \right)$$

64. Find  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial \theta}$  if  $w = \log(x^2 + y^2)$

$$\text{where } x = r \cos \theta, y = r \sin \theta$$

## 7. Integral Calculus and its applications

65. Evaluate the following problems using second fundamental

$$\text{theorem (i) } \int_0^{\frac{\pi}{2}} \frac{\sin x}{9+\cos^2 x} dx. \text{ (ii) } \int_0^{\frac{\pi}{2}} e^{2x} \cos x dx.$$

66. Evaluate (i)  $\int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx$ . (ii)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\tan x} dx$ .

67. Evaluate: (i)  $\int \sin^6 x dx$ . (ii)  $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x dx$ .

68. Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

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69. Find the area of the circle whose radius is  $a$  by using integral method.

70. Derive the formula for the volume of a right circular cone with radius  $r$  and height  $h$ .

71. Find the perimeter of the circle with radius  $a$  by using integral method.

72. Prove that the curved surface area of a sphere of radius  $r$  intercepted between two parallel planes at a distance  $a$  and  $b$  from the centre of the sphere is  $2\pi r(b-a)$  and hence deduct the surface area of the sphere ( $b > a$ ).



73. Find the volume of the solid that results when the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   $a > b > 0$  is revolved about the minor axis.

### 8. Differential Equations

74. Find the differential equation that will represent the family of all circles having centres on the x-axis and the radius is unity.
75. Form the differential equation from the following equations  
(i)  $y = A\cos 3x + B\sin 3x$ ,  $[A, B]$  (ii)  $y = Ae^{2x} + Be^{5x}$ ,  $[A, B]$
76. Solve: (i)  $(x + y)^2 \frac{dy}{dx} = a^2$  (ii)  $\frac{dy}{dx} = \sin(x + y)$
77. Solve: (i)  $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$  (ii)  $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$
78. The normal lines to a given curve at each point  $(x, y)$  on the curve pass through the point  $(2, 0)$ . The curve passes through the point  $(2, 3)$ . Formulate the differential equation representing the problem and hence find the equation of the curve.
79. Solve :  $(D^2 - 3D + 2) y = x$
80. Solve: (i)  $(1 + x^2) \frac{dy}{dx} + 2xy = \cos x$  (ii)  $\frac{dy}{dx} + y \cot x = 2 \cos x$
81. Find the equation of the curve passing through  $(1, 0)$  and which has slope  $1 + \frac{y}{x}$  at  $(x, y)$
82. Solve: (i)  $(D^2 + 9) y = \sin 3x$  (ii)  $(3D^2 + 4D + 1) y = 3e^{\frac{-x}{3}}$
83. In a certain chemical reaction the rate of conversion of a substance at time  $t$  is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour, 60 grams remain and at the end of 4 hours 21 grams. How many grams of the substance was there initially?

84. Find the cubic polynomial in  $x$  which attains its maximum value 4 and minimum value 0 at  $x = -1$  and 1 respectively.

### 9.DISCRETE MATHEMATICS

85. Construct the truth table for the following statements  
(i)  $\sim ((\sim p) \wedge (\sim q))$  (ii)  $(p \wedge q) \vee [\sim (p \wedge q)]$
86. Show that  $((\sim p) \vee (\sim q))$  is a tautology.
87. Show that  $((\sim q) \wedge p) \wedge q$  is a contradiction.
88. Show that  $p \rightarrow q \equiv (\sim p) \vee q$
89. Show that  $p \rightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
90. Show that  $(p \wedge q) \rightarrow (p \vee q)$
91. Construct the truth table for  $(p \wedge q) \vee (\sim r)$
92. Show that the cube roots of unity forms a finite abelian group under multiplication.
93. Prove that the set of all 4th roots of unity forms an abelian group under multiplication.
94. Write down and prove that Cancellation laws.
95. Show that the set  $[1], [3], [4], [5], [9]$  forms an abelian group under multiplication modulo 11.
96. Find the order of each element in the group  
 $(\mathbb{Z}_5 - [0], .5)$

**Best wishes by M.THIRUPATHYSATHIYA M.Sc.,M.Phil.,B.Ed.,CCA.,**

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### 10.PROBABILITY DISTRIBUTIONS

97. (i) Write down the properties of distribution function  
 (ii) Define Poisson distribution.write down the examples of Poisson distribution.
98. In a Binomial distribution  
 if  $n = 5$  and  $P(X = 3) = 2P(X = 2)$  find  $p$
99. Find the expected value of the number on a die when thrown.
100. Find the probability distribution of the number of sixes in throwing three dice once.
101. A discrete random variable X has the following probability distributions.
- |       |   |    |    |    |    |     |     |     |     |
|-------|---|----|----|----|----|-----|-----|-----|-----|
| X :   | 0 | 1  | 2  | 3  | 4  | 5   | 6   | 7   | 8   |
| P(X): | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |
- (i) Find the value of a (ii) Find  $P(x < 3)$   
 (iii) Find  $P(3 < x < 7)$
102. For the p.d.f
- $$f(x) = \begin{cases} cx(1-x)^3, & 0 < x < 1; \\ 0, & elsewhere \end{cases}$$
- find (i) the constant  $c$  (ii)  $P(X > \frac{1}{2})$
103. Find the Mean and Variance for the following probability density function  $f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & elsewhere \end{cases}$
104. Prove that the total probability is one.
105. In a Poisson distribution if  $P(X = 2) = P(X = 3)$  find  $P(X = 5)$  [ $e^{-3} = 0.050$ ]
106. Write down the properties of normal distribution.

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107. If  $X$  is normally distributed with mean 6 and standard deviation 5 find (i)  $P(0 \leq x \leq 8)$  (ii)  $P(|X - 6|) < 10$
108. Find the probability distribution of the number of sixes in throwing three dice once.
109. A die is tossed twice. A success is getting an odd number on a toss. Find the mean and the variance of the probability distribution of the number of successes.
110. If on an average 1 ship out of 10 do not arrive safely to ports. Find the mean and the standard deviation of ships returning safely out of a total of 500 ships.

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KUNICHI, TIRUPATTUR, VELLORE DISTRICT.

XII STD

Chapter 3

Dec 1, 2011

Time : 1.30 hrs

Complex Numbers

Max Marks: 100

**Section A**

$5 \times 6 = 30$  marks

**Answer any five Questions**

1. If  $z_1 = 2 + i$ ,  $z_2 = 3 - 2i$  and  $z_3 = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$

Find the conjugate of (i)  $z_1 z_2$  (ii)  $(z_3)^4$

2. Find the real values of  $x$  and  $y$  for  $\sqrt{x^2 + 3x + 8} + (x + 4)i = y(2 + i)$

3. If  $z_1$  and  $z_2$  be two complex numbers. then Prove

that (a)  $|z_1 + z_2| \leq |z_1| + |z_2|$  (b)  $|z_1 - z_2| \geq |z_1| - |z_2|$

4. For any two complex numbers  $z_1$  and  $z_2$  then

Prove that (a)  $|z_1 z_2| = |z_1| |z_2|$  (b)  $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

5. If  $\arg(z - 1) = \frac{\pi}{6}$  and  $\arg(z + 1) = \frac{2\pi}{3}$  then

Prove that  $|z| = 1$

6. Find the square root of (a)  $(-8 - 6i)$  (b)

$(-7 + 24i)$

7. Solve (a)  $x^9 + x^5 - x^4 - 1 = 0$  (b)  $x^4 - x^3 + x^2 - x + 1 = 0$

8. Find the fourth roots of unity and Prove that  $G.P$

9. Prove that  $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$

(OR)

10. Solve the equation  $x^4 - 8x^3 + 24x^2 - 32x + 20 = 0$

if  $3 + i$  is a root.

Section B

7 × 10 = 70 marks

Answer any seven Questions

11. P represents the variable complex number z. Find the locus of P, if  $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$

12. Prove that the solutions of any complex number always have two roots.

13. If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 2x + 2 = 0$  and

$$\cot\theta = y + 1$$

Show that 
$$\frac{(y + \alpha)^n - (y + \beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n\theta}$$

14. If  $a = cis2\alpha, b = cis2\beta, c = cis2\gamma$  then P.T

(i)  $\sqrt{abc} + \frac{1}{\sqrt{abc}} = 2\cos(\alpha + \beta + \gamma)$  (ii)  $\frac{a^2b^2 + c^2}{abc} = 2\cos2(\alpha + \beta - \gamma)$

15. Show that the points representing the complex numbers  $2i, (1 + i), (4 + 4i), (3 + 5i)$  in the Argand diagram respectively.

16. If  $x + \frac{1}{x} = 2\cos\theta$  and  $y + \frac{1}{y} = 2\cos\phi$  show that

(i)  $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$  (ii)  $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\theta - n\phi)$

17 Find all the values of  $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$  and hence prove that the product of the values is 1. (OR)

18. If  $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$  then

P.T (i)  $\cos2\alpha + \cos2\beta + \cos2\gamma = 0$

(ii)  $\sin2\alpha + \sin2\beta + \sin2\gamma = 0$

(iii)  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{3}{2}$

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XII STD

NOV 26, 2011

Time : 1.30 hrs

CHAPTER 5

Max Marks:

100

DIFFERENTIAL CALCULUS APPLICATIONS

-I

Section A

$5 \times 6 = 30$  marks

Answer any five Questions

1. Verify Rolle's theorem for the function  $f(x) = 4x^3 - 9x, -3/2 \leq x \leq 3/2$
2. Verify Lagrange's Law of mean for the function  
 $f(x) = x^3 - 5x^2 - 3x, [1, 3]$
3. Obtain the Maclaurin's Series for  $\tan x, -\pi/2 \leq x \leq \pi/2$ .
4. Obtain the Maclaurin's Series for  $\log_e(1 + x)$ .
5. Evaluate  $\lim_{x \rightarrow \infty} \frac{\sin(2/x)}{1/2}$ .
6. Evaluate (a)  $\lim_{x \rightarrow 0^+} \frac{\cot x}{\cot 2x}$  (b)  $\lim_{x \rightarrow 0^+} x^x$ .
7. Find the intervals on which  $f$  is increasing or decreasing  $\sin^4 x + \cos^4 x, [0, \pi/2]$ .
8. Find the local maximum and minimum values of the f function  $f(x) = 2x^3 + 5x^2 - 4x$
9. Find two numbers whose sum is 100 and whose product is a maximum.
10. Resistance to motion  $F$  of a moving vehicle is given by  $F = 5/x + 100x$

11. Find the intervals of concavity and the points of inflection of the function  $y = 12x^2 - 2x^3 - x^4$
12. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius  $r$ .
13. Find the Maclaurin's series expansion for  $\tan^{-1}(x)$
14. Show that the equation of the normal to the curve  $x = a\cos\theta; y = a\sin^3\theta$  at ' $\theta$ ' is  $x\cos\theta - y\sin\theta = a\cos 2\theta$
15. If the curve  $y^2 = x$  and  $xy = k$  are orthogonal then prove that  $8k^2 = 1$
16. Let P be a point on the curve  $y = x^3$  and suppose that the tangent line at P intersects the curve again at Q. Prove that the slope at Q is four times the slope at P.
- 17 Gravel is being dumped from a conveyor belt at a rate of  $30\text{ft}^3/\text{min}$  and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is  $10\text{ft}$  height? (OR)
18. At noon, ship A is  $100\text{km}$  west of ship B. Ship A is sailing east at  $35\text{km/hr}$  and Ship B is sailing north at  $25\text{km/hr}$ . How fast is the distance between the ships changing at  $4.00\text{p.m}$



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CHAPTER 8

Max Marks:

100

Differential equations

Section A

$5 \times 6 = 30$  marks

Answer any five Questions

1. Solve  $\frac{dy}{dx} = \sin(x + y)$ .

2. Solve  $(x + y)^2 \frac{dy}{dx} = 1$ .

3. Solve  $x^2 \frac{dy}{dx} = y^2 + 2xy$ . when  $y = 1$ , when  $x = 1$

4. Show that the equation of the curve whose slope at any point is equal to  $y + 2x$  and which passes through the origin is  $y = 2(e^x - x - 1)$ .

5. Solve  $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ .

6. Form the differential equation by eliminating arbitrary constants

(a)  $y = e^x(C \cos 2x + D \sin 2x)$       (b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ .

7. solve  $(1 + x^2) \frac{dy}{dx} + 2xy = \cos x$ .

8. Solve  $(D^2 + 2D + 3)y = \sin 2x$

9. Solve  $(D^2 - 6D + 9)y = x + e^{2x}$ .

10. Solve  $(D^2 - 2D - 3)y = \sin x \cos x$

Section B

$7 \times 10 = 70$  marks

Answer any seven Questions

11. Solve  $(D^2 - 3D + 2)y = 2e^{3x}$ , when  $x = \log 2, y = 0$   
and when  $x = 0, y = 0$
12. Solve  $(D^2 + 1)y = 0$ . when  $x = 0, y = 2$   
and when  $x = \pi/2, y = -2$
13. Solve  $\frac{dy}{dx} = \frac{y(x - 2y)}{x(x - 3y)}$ .
14. Solve  $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$ .  
given that  $y = 1$ , where  $x = 0$ .
15. Solve  $\frac{dy}{dx} + \frac{x}{1 + y^2} = \frac{\tan^{-1}y}{1 + y^2}$ .
16. Random disappears at a rate proportional to the amount present. If 5 percentage of the original amount disappears in 50 years ,how much will remain at the end of 100 years. [Take  $A_0$  as the initial amount.]
- 17 The rate at which the population of a city increases at any time is proportional to the population at that time.If there were 1, 30, 000 people in the city in 1960 and 1, 60, 000 in 1990 what population may be anticipated in 2020. OR
18. Solve (a) $(D^2 - 4D + 1)y = x^2$       (b) $(D^2 + 4D + 13)y = \cos 3x$

#### 4. Analytical Geometry-Practical Problems

$10 \times 10 = 100$  marks

1. The girder of a railway bridge is in the parabolic form with span 100 ft. and the highest point on the arch is 10 ft. above the bridge. Find the height of the bridge at 10 ft. to the left or right from the midpoint of the bridge.
2. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.
3. Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

4. A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of  $\frac{\pi}{3}$  radians with the axis of the orbit. find (i) the equation of the comets orbit (ii) how close does the comet come nearer to the sun? (Take the orbit as open rightward).
5. A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of the cable on the towers are 200ft above the road way and the lowest point on the cable is 70ft above the roadway. Find the vertical distance to the cable (parallel to the roadway) from a pole whose height is 122 ft.
6. A cable of a suspension bridge is in the form of a parabola whose span is 40 mts. The road way is 5 mts below the lowest point of the cable. If an extra support is provided across the cable 30 mts above the ground level, find the length of the support if the height of the pillars are 55 mts.

7. The ceiling in a hallway 20 ft wide is in the shape of a semi-ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12ft.
8. A ladder of length 15m moves with its ends always touching the vertical wall and the horizontal floor. Determine the equation of the locus of a point P on the ladder, which is 6 m from the end of the ladder in contact with the floor.
9. A kho-kho player in a practice session while running realises that the sum of the distances from the two kho-kho poles from him is always 8 m. Find the equation of the path traced by him if the distance between the poles is 6m.
10. A satellite is travelling around the earth in an elliptical orbit having the earth at a focus and of eccentricity  $\frac{1}{2}$ . The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.

Best wishes by M.THIRUPATHY M.Sc.,M.Phil.,B.Ed.,CCA.,

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